GRAPH THEORY MID TERM EXAM

This exam is of 40 marks. There are 8 questions Please do not cheat. Good luck! (40)

Notation

- $\nu = \nu(G)$ - no. of vertices. $e = e(G)$ - no. of edges. $w = w(G)$ - no. of components.
- $d(v)$ - degree of a vertex. $\delta = \delta(G)$ - smallest degree. $\Delta = \Delta(G)$ - largest degree.
- $\tau(G)$ - number of spanning trees.
- $\kappa(G) = \kappa$ - vertex connectivity of G. $\kappa'(G) = \kappa'$ - edge connectivity of G.

1. The complement G^c of a simple graph G is the complement of the graph in the complete graph with the same number of vertices. G is *self-complementary* if $G^c \simeq G$. Show

(a) If G is self-complementary then $\nu(G) \equiv 0$ or $1 \bmod 4$. (4)
(b) Give an example of a non-trivial self-complementary graph. (3)

2. Show that if $\delta > 2$ then G contains a cycle. (3)

3. Show that if G is a graph, G contains at least $e - \nu + w$ distinct cycles. (4)

4. Recall that $\tau(K_n) = n^{n-2}$. Show that if e is an edge of K_n then (5)

$$\tau(K_n - e) = (n-2)n^{n-3}.$$

5. (a) Show that if G is simple and $\delta \geqslant \nu/2$ then $\kappa' = \delta$. (3)
5. (b) Find a simple graph G with $\delta = [(\nu/2) - 1]$ and $\kappa' < \delta$. (3)

6. Recall that a *block* is a connected graph with no cut vertices and a *block of a graph* is a subgraph of G which is maximal with respect to this property. Show that the number of blocks of a graph G is

$$w(G) + \sum_{\nu \in V(G)} (b(\nu) - 1)$$

where $b(\nu)$ denotes the number of blocks of G containing ν. (5)

7. Construct, or disprove the existence of, an Eulerian graph with $\nu(G)$ even and $e(G)$ odd. (5)

8. Show that if G has a Hamiltonian *path* then, for every proper subset S of V, (5)

$$w(G - S) \leqslant |S| + 1.$$

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